ROYAL CIVIL SERVICE COMMISSION
BHUTAN CIVIL SERVICE EXAMINATION (BCSE) 2020
EXAMINATION CATEGORY: TECHNICAL

PAPER III: SUBJECT SPECIALISATION PAPER FOR STATISTICS

Date : February 27, 2021
Total Marks : 100
Writing Time : 150 minutes (2.5 hours)
Reading Time : 15 minutes (prior to writing time)

GENERAL INSTRUCTIONS:

1. Write your Registration Number clearly and correctly on the Answer Booklet.
2. The first 15 minutes is to check the number of pages of the Question Paper, printing errors, clarify doubts and to read the instructions. You are NOT permitted to write during this time.
3. This paper consists of **TWO SECTIONS**, namely SECTION A & SECTION B:
    - **SECTION A** has two parts: Part I - 30 Multiple Choice Questions
      Part II - 4 Short Answer Questions

      All questions under SECTION A are COMPULSORY.
    - **SECTION B** consists of two Case Studies. Choose only **ONE** case study and answer the questions of your choice.
4. All answers should be written on the Answer Booklet provided to you. Candidates are not allowed to write anything on the question paper. If required, ask for additional Answer Booklet.
5. All answers should be written with correct numbering of Section, Part and Question Number in the Answer Booklet provided to you. Note that any answer written without indicating the Section, Part and Question Number will NOT be evaluated and no marks will be awarded.
6. Begin each Section and Part on a fresh page of the Answer Booklet.
7. You are not permitted to tear off any sheet(s) of the Answer Booklet as well as the Question Paper.
8. Use of any other paper including paper for rough work is not permitted.
9. **You must hand over the Answer Booklet to the Invigilator before leaving the examination hall.**
10. This paper has **14 printed pages**, including this instruction page.

GOOD LUCK!

SECTION A

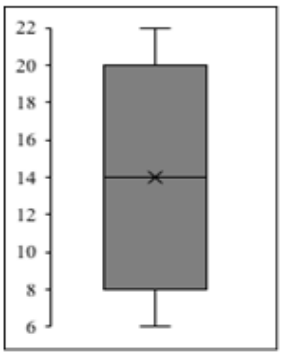
PART I: Multiple Choice Questions (30 marks)

Choose the correct answer and write down the letter of your chosen answer in the Answer Booklet against the question number e.g. 31 (d). Each question carries ONE mark. Any double writing, smudgy answers or writing more than one choice shall not be evaluated.

1. Variables can be categorical or numerical. Which one of the following is a categorical variable?
   a) Heights of basketball players.
   b) Age of basketball players.
   c) Educational attainment of basketball players.
   d) Gender of basketball players with 0 representing male and 1 representing female.

2. The test scores of 4 students in a statistics test are 20, 30, 15 and 75. What is the percentile rank of the student who scored 30?
   a) 80^{th} percentile
   b) 75^{th} percentile
   c) 45^{th} percentile
   d) 35^{th} percentile

Use the following box and whisker plot to answer **Question 3**. It represent the waiting time in minutes of a sample of 27 customers who called for help support from Royal Bhutan Police.

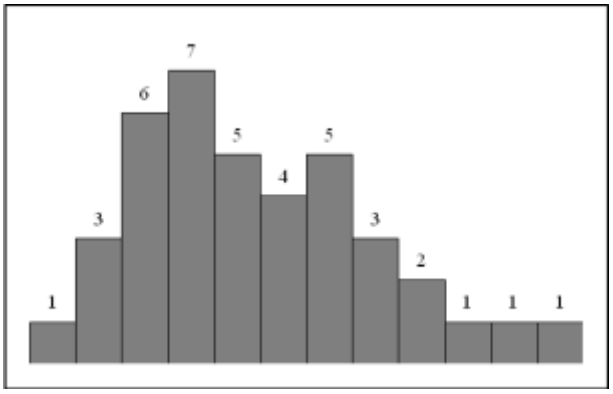


3. Which of the following statements is NOT true?
   a) About 50% of the time, these customers waited 14 minutes or more.
   b) The mean waiting time for these customers is 14 minutes.
   c) The range of the waiting time for this group of customers is 16 minutes.
   d) The distribution of waiting times for this group of customers is symmetric.

4. In a set of 15 observations, the mean is equal to 27. A value of 43 is added to the dataset. What is the new mean of these 16 observations?
   a) 27
   b) 28
   c) 29
   d) 30

5. The median of a dataset is 36. One value of the dataset was mistakenly listed as 42 when it should have been 82. What is the new median after the correction is made?
   a) 46
   b) 36
   c) 26
   d) 16

6. A histogram of a set of 39 observations is shown below with frequency listed above each bar.



   Which of the following is NOT true about the distribution?
   a) The mean is greater than median and the distribution is skewed to the right.
   b) The range of the distribution is 6.
   c) The mode of the distribution is 1.
   d) The distribution has outliers.

7. If all values in a dataset are exactly the same, what is the standard deviation?
   a) 0
   b) 1
   c) Less than 0
   d) Greater than 1

Use the stem-leaf-plot below to answer **Question 8**. It shows the test scores for 30 students.

6| 58
7| 011
7| 5699
8| 122344
8| 5566788
9| 0123
9| 5589

8. How do you interpret the percentile for the student who scored 92?
   a) The student who scored 92 is higher than approximately 85% of the scores.
   b) The student who scored 92 is higher than approximately 83% of the scores.
   c) The student who scored 92 is higher than approximately 80% of the scores.
   d) The student who scored 92 is higher than approximately 75% of the scores.

9. Which of the following is a negatively skewed distribution?
   a) Mean=5, median=6, mode=7
   b) Mean=6, median=5, mode=7
   c) Mean=5, median=5, mode=7
   d) Mean=5, median=5, mode=5

10. For most large-scale studies, it is impossible to study the entire population. Statisticians or researchers typically gather a sample, which is a subset of population and then study the sample to draw inferential statistics about population. This type of study method is called
    a) Census
    b) Survey
    c) Mixed of census and survey
    d) None of the above

11. Which of the following is TRUE about the 95% confidence level?
    a) A confidence level of 95% means that if we repeat the experiment numerous times (under the same conditions), the results will match that of the population in 95% of all possible cases.
    b) A confidence level of 95% means that if we repeat the experiment numerous times (under the same conditions), the results will match that of the population in 5% of all possible cases.
    c) A confidence level of 95% means that if we repeat the experiment numerous times (under the same conditions), the probability that you will get the results will be 95%.
    d) None of the above.

Use the following information to answer **Question 12 to 14.**

Ministry of Health (MoH) conducted drug tests of 10,000 individuals in 2019. The summary statistics of the test results are provided below.

	Tested positive	**Tested Negative**	**Total**
Drug user	495	5	500
Non-user	95	9,405	9,500
Total	**590**	**9,410**	**10,000**

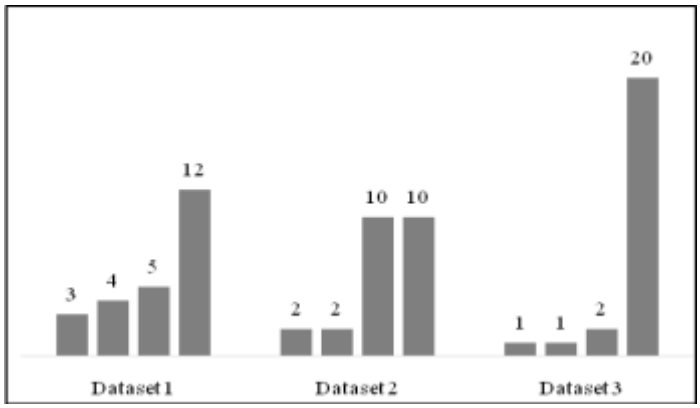
12. What is the probability that an individual with a positive test result is an actual drug user?
    a) 0.01
    b) 0.16
    c) 0.84
    d) 0.99

13. What is the probability that an individual with a positive test result is a non-drug user (false-positive)?
    a) 0.01
    b) 0.16
    c) 0.84
    d) 0.99

14. What is the probability that an individual with a negative test result is an actual drug user (false-negative)?
    a) 0.010
    b) 0.05
    c) 0.95
    d) 0.99

15. The mean and median of the dataset: 90, 89, 84, 80, 77, 70, 70, 59, 55, 33, 32, 31, 30 is 64 and 70 respectively. It was observed later that instead of 30, the number is 24. What is the new mean and median?
    a) Mean=64 and median=70
    b) Mean=64 and median=70.5
    c) Mean=63.5 and median=70
    d) Mean=63.5 and median=70.5

Use the following bar graph of three datasets to answer **Question 16.**



16. Based on the information provided in the bar graph, which of the following is NOT true about the Dataset 1, Dataset 2 and Dataset 3?
    a) Dataset 3 is more variable compared to Dataset 1 and Dataset 2.
    b) Dataset 1 has less variation compared to Dataset 2 and Dataset 3.
    c) Dataset 1, Dataset2 and Dataset 3 have same mean.
    d) Dataset 1 and Dataset 2 have a similar range.

17. A total of 100 students have a mean study time of 22 hours and a standard deviation of 5.7. What would be the confidence intervals for 95% confidence with an alpha of 0.05?
    a) (20.88, 23.12)
    b) (21.43, 22.57)
    c) (22.00, 27.70)
    d) (22.00, 28.00)

Use the following table to answer **Question 18**. It's the summary statistics for maths and statistics tests by students.

	Mean	**Standard deviation**
Maths	$\mu = 55$	$\sigma = 10$
Statistics	$\mu = 65$	$\sigma = 20$

18. Sonam took both the exams and scored 75 in maths, 85 in statistics. Which exam did he do relatively better?
    a) Statistics exam
    b) Maths exam
    c) Equally good in both the exams
    d) None of the above

19. The probability P(Z>0) is 0.5. What can you say about the variable Z?
    a) Z is a standard normal variable.
    b) Z is not a standard normal variable.
    c) Z is not a variable at all.
    d) None of the above.

Use the following information to answer **Questions 20 to 25.**

The heights of students who appeared for RCSC Preliminary Exam (PE) are normally distributed with a mean of 150 cm and a standard deviation of 20 cm.

20. Jigme with a height of 161.4 cm was selected at random. What proportions of students' height are lower than Jigme's height?
    a) 0.7054
    b) 0.7088
    c) 0.7123
    d) 0.7157

21. Norbu with a height of 170 cm was also selected at random. What proportions of students' heights are higher than his height?
    a) 0.8413
    b) 0.8438
    c) 0.1587
    d) 0.1562

22. What proportions of students' heights are between 90 cm and 210 cm?
    a) 0.9968
    b) 0.9969
    c) 0.9970
    d) 0.9971

23. What proportions of students' heights are above 210 cm?
    a) 0.0015
    b) 0.0030
    c) 0.0035
    d) 0.0040

24. A normal distribution with mean 0 and standard deviation 1 is called
    a) Standard normal distribution.
    b) Binomial distribution.
    c) Poisson distribution.
    d) None of the above.

25. Which of the following is TRUE about the standard normal distribution?
    a) The total area under the standard normal distribution curve is 100.
    b) The total area under the standard normal distribution curve is 1.
    c) The total area under the standard normal distribution curve is 0.50
    d) The total area under the standard normal distribution curve is 0.25

26. An approximately normal distribution has a mean of 48 and a standard deviation of 6. Without using the calculator, what percentage of the distribution is between 36 and 60?
    a) 68%
    b) 95%
    c) 97%
    d) 99%

Use the following information to answer **Questions 27 to 28**

The summary statistics of a survey of 67 students who appeared in Statistics paper are provided below:

N	Mean	SD	Min	Q1	Median	Q3	Max
67	4.0996	0.7055	2.92	3.65	3.97	4.35	6.72

27. Given the information above, which of the following is TRUE?
    a) It is reasonable to believe that the distribution is skewed towards the right.
    b) It is reasonable to believe that the distribution is skewed towards the left.
    c) It is reasonable to believe that the distribution is not normally distributed.
    d) It is reasonable to believe that the distribution is approximately normally distributed.

28. Using the same information above, which of the following observations is NOT true?
    a) The distribution has a centre of 4.09 with a spread of 0.71.
    b) The distribution has a Co-efficient of Variation (CV) approximately 50%.
    c) The distribution has at least one outlier at the right because 6.72 is greater than Q3+1.5*(IQR).
    d) The distribution has at least one outlier at the left because 2.92 is less than Q1-1.5*(IQR).

29. To measure the average height of an apple tree, a researcher divides a large plot of mango orchard into hundreds of 10-meter-by-10-meter plots. The researcher then chooses 10 of these plots at random and measures all of the heights of apple trees in those plots. Which of the following represents the sampling plan?
    a) Stratified random sampling
    b) Simple random sampling
    c) Systematic sampling
    d) Cluster sampling

30. Which one of the following equation represents a relatively strong negative linear relationship?
    a) r= 0.23, y=16.5332-0.4451x
    b) r= -0.93, y=16.5332+0.4451x
    c) r= 0.93, y=16.5332+0.4451x
    d) r= -0.83, y=16.5332-0.4451x

PART II – Short Answer Questions [20 marks]

This part has 4 Short Answer Questions. Answer ALL the questions. Each question carries 5 marks.

1. How do you define and differentiate between the following?
    a) Data and variable
    b) Nominal and ordinal scale
    c) Parameters and statistics

Use the following information to answer **Questions 2 to 4.**

Consider the following two datasets:

Set 1: 1,2,3,4,5,6,6,7,8,9,10,11

Set 2: 4,5,5,5,6,6,6,6,7,7,7,8

2. What is the mean, median and mode of each of the two datasets?

3. What is the main difference between these two datasets? Apply statistical method to check the main difference between these two datasets.

4. Are there any outliers in any of these datasets? Apply statistical method to check for data outliers.

SECTION B: CASE STUDY [50 marks]

Choose either CASE I or CASE II from this section. Each case study carries 50 marks. Mark for each sub-question is indicated in the brackets.

CASE I

The table below shows the observations and summary statistics of weight loss for individuals who were on different diets (A, B, C & D).

Diet	Weight loss	Sample mean	Sample SD
A	9.9, 9.6, 8.0, 4.9, 10.2	9.18	2.29
B	9.5, 3.8, 11.5, 9.2, 4.5	8.91	2.78
C	10.8, 10.2, 14.4, 8.7, 12.2	12.11	1.79
D	13.2, 9.5, 10.6, 9.9, 9.5	10.54	2.23

Using the information provided in the table above, you as a statistician is given an assignment to check whether there is difference in the mean weight loss for individuals who were on different diets. Based on the information provided, you will do **ANOVA (Analysis of Variance) test**.

1. What would be your null and alternative hypothesis? (5 marks)

2. Calculate and provide missing information in the table below. An example of the missing information for Diet A are provided. (10 marks)

Diet	No. of observations	Sum	Average	Variance
Example A	*5*	*42.6*	*8.52*	*4.82*
B				
C				
D				

3. Calculate the following (30 marks)

Source of variation	Sum of Square (SS)	Degrees of freedom (df)	Mean square (MS)	F-stat	F-critical	P-value
Between Group						*0.11*
Within Group						
Total						

4. At $\alpha = 0.05$ level of significance, is there sufficient evidence to conclude that the mean weight loss differs by diets? (5 marks)

CASE II

Table below shows the production of potatoes by dzongkhags in 2018 and 2019 according to the Ministry of Agriculture & Forests. The summary statistics (mean, sd and n) are also provided for easy reference.

Dzongkhag	**2018**	**2019**
Bumthang	3,926.07	4,007.93
Chhukha	2,515.55	2,420.65
Dagana	140.96	283.39
Gasa	118.58	185.24
Haa	2,267.68	1,512.00
Lhuntse	711.47	593.20
Monggar	3,235.33	3,171.93
Paro	4,661.30	4,246.45
Pema Gatshel	1,236.13	1,356.54
Punakha	154.93	173.71
SamdrupJongkhar	536.11	1,102.89
Samtse	226.12	433.88
Sarpang	197.10	364.76
Thimphu	1,820.06	1,470.04
Trashigang	4,400.06	4,222.53
Trashi Yangtse	1,697.88	1,476.45
Trongsa	448.72	627.24
Tsirang	219.87	265.78
WangduePhodrang	15,661.85	15,569.87
Zhemgang	102.24	75.78
Mean	**2,214**	**2,178**
SD	**3,514**	**3,452**
n	**20**	**20**

Using the above information, you will conduct two-sample t-test with unequal variance whether mean production of potatoes differs by years at $\alpha=0.05$ *level of significance*. Answer the following questions:

1. What would be your null and alternative hypothesis? (10 marks)

2. What is the value of t-statistics? (10 marks)

3. What is the approximate p-value? [hint: refer t-distribution table] (20 marks)

4. At $\alpha=0.05$ level of significance, is there sufficient evidence to conclude that the mean production of potatoes differ by years? (10 marks)

LIST OF STATISTICAL FORMULA

1. $\overline{x} = \frac{\sum_{i=1}^{n} x_i}{n}$
2. $s^2 = \frac{\sum_{i=1}^{n}(x_i - \overline{x})^2}{(n-1)}$
3. $\mu = \frac{\sum_{i=1}^{n} X_i}{N}$
4. $\sigma^2 = \frac{\sum_{i=1}^{n}(x_i - \mu)^2}{N}$
5. $\mu = E(x) = \sum_{i=1}^{N} x_i P(x_i)$
6. $\sigma^2 = V(x) = \sum_{i=1}^{N}(x_i - \mu)^2 P(x_i) = \sum_{i=1}^{N} {x_i}^2 P(x_i) - \mu^2$
7. $Coefficient\ of\ variation\ (CV) = \frac{SD}{\overline{x}} * 100$
8. $x \sim Normally\ (\mu, \sigma^2)$
9. $z = \frac{x-\mu}{\sigma}$
10. $\overline{x} \pm z_{\frac{\alpha}{2}} \frac{\sigma}{\sqrt{n}}$
11. $\overline{x} \pm t_{\frac{\alpha}{2},(n-1)} \frac{s}{\sqrt{n}}$
12. $n = \frac{z^2 \frac{\alpha}{2} \sigma^2}{B^2}$
13. $r = \frac{\sum_{i=1}^{n}(x-\overline{x})(y-\overline{y})}{\sqrt{(x-\overline{x})^2 (y-\overline{y})^2}}$
14. $t - stat = \frac{\overline{x}_A - \overline{x}_B}{\sqrt{\frac{{s_A}^2}{n_A} + \frac{{s_B}^2}{n_B}}}$

Table of Normal Curve Areas

z	0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998
3.5	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998	0.9998
3.6	0.9998	0.9998	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.7	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999
3.8	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999	0.9999

Table of t-distribution critical values

Upper tail probability values												
df	**0.25**	**0.2**	**0.15**	**0.1**	**0.05**	**0.025**	**0.02**	**0.01**	**0.005**	**0.0025**	**0.001**	**0.0005**
1	1.000	1.376	1.963	3.078	6.310	12.700	15.900	31.820	63.650	127.300	318.300	636.619
2	0.817	1.061	1.386	1.886	2.920	4.303	4.849	6.965	9.925	14.080	22.330	31.599
3	0.765	0.979	1.250	1.638	2.353	3.182	3.482	4.541	5.841	7.453	10.220	12.924
4	0.741	0.941	1.190	1.533	2.132	2.776	2.999	3.747	4.604	5.598	7.173	8.610
5	0.727	0.920	1.156	1.476	2.015	2.571	2.757	3.365	4.032	4.773	5.893	6.869
6	0.718	0.906	1.134	1.440	1.943	2.447	2.612	3.143	3.707	4.317	5.208	5.959
7	0.711	0.896	1.119	1.415	1.895	2.365	2.517	2.998	3.499	4.029	4.785	5.408
8	0.706	0.889	1.108	1.397	1.860	2.306	2.449	2.896	3.355	3.833	4.501	5.041
9	0.703	0.883	1.100	1.383	1.833	2.262	2.398	2.821	3.250	3.690	4.297	4.781
10	0.700	0.879	1.093	1.372	1.812	2.228	2.359	2.764	3.169	3.581	4.144	4.587
11	0.697	0.876	1.088	1.363	1.796	2.201	2.328	2.718	3.106	3.497	4.025	4.437
12	0.696	0.873	1.083	1.356	1.782	2.179	2.303	2.681	3.055	3.428	3.930	4.318
13	0.694	0.870	1.079	1.350	1.771	2.160	2.282	2.650	3.012	3.372	3.852	4.221
14	0.692	0.868	1.076	1.345	1.761	2.145	2.264	2.624	2.977	3.326	3.787	4.140
15	0.691	0.866	1.074	1.341	1.753	2.131	2.249	2.602	2.947	3.286	3.733	4.073
16	0.690	0.865	1.071	1.337	1.746	2.120	2.235	2.583	2.921	3.252	3.686	4.015
17	0.689	0.863	1.069	1.333	1.740	2.110	2.224	2.567	2.898	3.222	3.646	3.965
18	0.688	0.862	1.067	1.330	1.734	2.101	2.214	2.552	2.878	3.197	3.610	3.922
19	0.688	0.861	1.066	1.328	1.729	2.093	2.205	2.539	2.861	3.174	3.579	3.883
20	0.687	0.860	1.064	1.325	1.725	2.086	2.197	2.528	2.845	3.153	3.552	3.850
21	0.686	0.859	1.063	1.323	1.721	2.080	2.189	2.518	2.831	3.135	3.527	3.819
22	0.686	0.858	1.061	1.321	1.717	2.074	2.183	2.508	2.819	3.119	3.505	3.792
23	0.685	0.858	1.060	1.319	1.714	2.069	2.177	2.500	2.807	3.104	3.485	3.768
24	0.685	0.857	1.059	1.318	1.711	2.064	2.172	2.492	2.797	3.091	3.467	3.745
25	0.684	0.856	1.058	1.316	1.708	2.060	2.167	2.485	2.787	3.078	3.450	3.725
26	0.684	0.856	1.058	1.315	1.706	2.056	2.162	2.479	2.779	3.067	3.435	3.707
27	0.684	0.855	1.057	1.314	1.703	2.052	2.158	2.473	2.771	3.057	3.421	3.690
28	0.683	0.855	1.056	1.313	1.701	2.048	2.154	2.467	2.763	3.047	3.408	3.674
29	0.683	0.854	1.055	1.311	1.699	2.045	2.150	2.462	2.756	3.038	3.396	3.659
30	0.683	0.854	1.055	1.310	1.697	2.042	2.147	2.457	2.750	3.030	3.385	3.646
40	0.681	0.851	1.050	1.303	1.684	2.021	2.123	2.423	2.704	2.971	3.307	3.551
50	0.679	0.849	1.047	1.299	1.676	2.009	2.109	2.403	2.678	2.937	3.261	3.496
60	0.679	0.848	1.045	1.296	1.671	2.000	2.099	2.390	2.660	2.915	3.232	3.460
80	0.678	0.846	1.043	1.292	1.664	1.990	2.088	2.374	2.639	2.887	3.195	3.416
100	0.677	0.845	1.042	1.290	1.660	1.984	2.081	2.364	2.626	2.871	3.174	3.390
1000	0.675	0.842	1.037	1.282	1.646	1.962	2.056	2.330	2.581	2.813	3.098	3.300
z*	0.674	0.841	1.036	1.282	1.645	1.960	2.054	2.326	2.576	2.807	3.090	3.291
	50%	**60%**	**70%**	**80%**	**90%**	**95%**	**96%**	**98%**	**99%**	**99.50%**	**99.80%**	**99.90%**
Confidence Level												

Table of f-distribution critical values for $\alpha = 0.05$

d2	d1																		
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
1	161.4	199.5	215.7	224.6	230.2	234	236.8	238.9	240.5	241.9	243.9	245.9	248	249.1	250.1	251.1	252.2	253.3	254.3
2	18.51	19	19.16	19.25	19.3	19.33	19.35	19.37	19.38	19.4	19.41	19.43	19.45	19.45	19.46	19.47	19.48	19.49	19.5
3	10.13	9.55	9.28	9.12	9.01	8.94	8.89	8.85	8.81	8.79	8.74	8.7	8.66	8.64	8.62	8.59	8.57	8.55	8.53
4	7.71	6.94	6.59	6.39	6.26	6.16	6.09	6.04	6	5.96	5.91	5.86	5.8	5.77	5.75	5.72	5.69	5.66	5.63
5	6.61	5.79	5.41	5.19	5.05	4.95	4.88	4.82	4.77	4.74	4.68	4.62	4.56	4.53	4.5	4.46	4.43	4.4	4.36
6	6.99	5.14	4.76	4.53	4.39	4.28	4.21	4.15	4.1	4.06	4	3.94	3.87	3.84	3.81	3.77	3.74	3.7	3.67
7	5.59	4.74	4.35	4.12	3.97	3.87	3.79	3.73	3.68	3.64	3.57	3.51	3.44	3.41	3.38	3.34	3.3	3.27	3.23
8	5.32	4.46	4.07	3.84	3.69	3.58	3.5	3.44	3.39	3.35	3.28	3.22	3.15	3.12	3.08	3.04	3.01	2.97	2.93
9	5.12	4.26	3.86	3.63	3.48	3.37	3.29	3.23	3.18	3.14	3.07	3.01	2.94	2.9	2.86	2.83	2.79	2.75	2.71
10	4.96	4.1	3.71	3.48	3.33	3.22	3.14	3.07	3.02	2.98	2.91	2.85	2.77	2.74	2.7	2.66	2.62	2.58	2.54
11	4.84	3.98	3.59	3.36	3.2	3.09	3.01	2.95	2.9	2.85	2.79	2.72	2.65	2.61	2.57	2.53	2.49	2.45	2.4
12	4.75	3.89	3.49	3.26	3.11	3	2.91	2.85	2.8	2.75	2.69	2.62	2.54	2.51	2.47	2.43	2.38	2.34	2.3
13	4.67	3.81	3.41	3.18	3.03	2.92	2.83	2.77	2.71	2.67	2.6	2.53	2.46	2.42	2.38	2.34	2.3	2.25	2.21
14	4.6	3.74	3.34	3.11	2.96	2.85	2.76	2.7	2.65	2.6	2.53	2.46	2.39	2.35	2.31	2.27	2.22	2.18	2.13
15	4.54	3.68	3.29	3.06	2.9	2.79	2.71	2.64	2.59	2.54	2.48	2.4	2.33	2.29	2.25	2.2	2.16	2.11	2.07
16	4.49	3.63	3.24	3.01	2.85	2.74	2.66	2.59	2.54	2.49	2.42	2.35	2.28	2.24	2.19	2.15	2.11	2.06	2.01
17	4.45	3.59	3.2	2.96	2.81	2.7	2.61	2.55	2.49	2.45	2.38	2.31	2.23	2.19	2.15	2.1	2.06	2.01	1.96
18	4.41	3.55	3.16	2.93	2.77	2.66	2.58	2.51	2.46	2.41	2.34	2.27	2.19	2.15	2.11	2.06	2.02	1.97	1.92
19	4.38	3.52	3.13	2.9	2.74	2.63	2.54	2.48	2.42	2.38	2.31	2.23	2.16	2.11	2.07	2.03	1.98	1.93	1.88
20	4.35	3.49	3.1	2.87	2.71	2.6	2.51	2.45	2.39	2.35	2.28	2.2	2.12	2.08	2.04	1.99	1.95	1.9	1.84
21	4.32	3.47	3.07	2.84	2.68	2.57	2.49	2.42	2.37	2.32	2.25	2.18	2.1	2.05	2.01	1.96	1.92	1.87	1.81
22	4.3	3.44	3.05	2.82	2.66	2.55	2.46	2.4	2.34	2.3	2.23	2.15	2.07	2.03	1.98	1.94	1.89	1.84	1.78
23	4.28	3.42	3.03	2.8	2.64	2.53	2.44	2.37	2.32	2.27	2.2	2.13	2.05	2.01	1.96	1.91	1.86	1.81	1.76
24	4.26	3.4	3.01	2.78	2.62	2.51	2.42	2.36	2.3	2.25	2.18	2.11	2.03	1.98	1.94	1.89	1.84	1.79	1.73
25	4.24	3.39	2.99	2.76	2.6	2.49	2.4	2.34	2.28	2.24	2.16	2.09	2.01	1.96	1.92	1.87	1.82	1.77	1.71
26	4.23	3.37	2.98	2.74	2.59	2.47	2.39	2.32	2.27	2.22	2.15	2.07	1.99	1.95	1.9	1.85	1.8	1.75	1.69
27	4.21	3.35	2.96	2.73	2.57	2.46	2.37	2.31	2.25	2.2	2.13	2.06	1.97	1.93	1.88	1.84	1.79	1.73	1.67
28	4.2	3.34	2.95	2.71	2.56	2.45	2.36	2.29	2.24	2.19	2.12	2.04	1.96	1.91	1.87	1.82	1.77	1.71	1.65
29	4.18	3.33	2.93	2.7	2.55	2.43	2.35	2.28	2.22	2.18	2.1	2.03	1.94	1.9	1.85	1.81	1.75	1.7	1.64
30	4.17	3.32	2.92	2.69	2.53	2.42	2.33	2.27	2.21	2.16	2.09	2.01	1.93	1.89	1.84	1.79	1.74	1.68	1.62
40	4.08	3.23	2.84	2.61	2.45	2.34	2.25	2.18	2.12	2.08	2	1.92	1.84	1.79	1.74	1.69	1.64	1.58	1.51
60	4	3.15	2.76	2.53	2.37	2.25	2.17	2.1	2.04	1.99	1.92	1.84	1.75	1.7	1.65	1.59	1.53	1.47	1.39
120	3.92	3.07	2.68	2.45	2.29	2.17	2.09	2.02	1.96	1.91	1.83	1.75	1.66	1.61	1.55	1.5	1.43	1.35	1.25
Infinity	3.84	3	2.6	2.37	2.21	2.1	2.01	1.94	1.88	1.83	1.75	1.67	1.57	1.52	1.46	1.39	1.32	1.22	1